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Investigating the Precision of the Mathematical Language of Secondary Teachers – A Small Sample Case Study

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ABSTRACT This paper explores the use of exact mathematical language of three secondary teachers by using the conceptual framework of language repertoires: literal, algebraic, graphical and procedural that relate to the various representations of a mathematical object. A repertoire in this study is an inventory of words as a cognitive resource. A questionnaire and two short interviews facilitated the data collection for this explorative mixed methods case study. The results indicate that teachers have a store of mathematical language that they can use in their teaching but they are not able to distinguish between the four language repertoires. Mathematical vocabularies could be classified under the four repertoires but the accuracy of vocabulary used within specific mathematical contexts was problematic. The findings of this study have implications for teaching and learning mathematics, for programmes designed to prepare students for secondary mathematics teaching, and for in-service teachers, and indirectly for learners of mathematics.

INTRODUCTION

In South Africa, the national senior certificate (NSC) mathematics results are very poor. Annual statistics show that 51.1 percent of NSC mathematics school leavers pass with thirty percent or more whilst the nine year average is 33.5 percent of those who sit the examination pass with forty percent or more (van Jaarsveld 2017). Far more than half of the NSC mathematics population about eighty percent of learners pass the subject with less than fifty percent. Motshekga, the South African minister of Basic Education, stated "the quality of education of any system is predictable on the quality of its teachers" (Department of Basic Education 2016). As such it is important for teachers to have the highest level of knowledge and understanding especially subject specialists. Evaluating teachers' skill sets regarding their grasp and use of mathematical language in the teaching and learning of mathematics may however be regarded as another dimension of quality, the rigour of which to this point has not been investigated.

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Mathematics education in South Africa has been in crisis for more than two decades. It has been widely acknowledged that both NSC mathematics results show no improvement and that South African mathematics teachers are poorly qualified or unqualified to teach the subject, but the nature of their qualifications is neither qualified nor quantified. That English is the primary language of instruction (De Klerk 2002; De Wet 2002; Kgosana 2006; Rademeyer 2006; Uys et al. 2007) may be one of the factors that aggravate an ongoing crisis.

Literature Review

Language of Teaching Mathematics

The focus of this paper is the use of an exact language of mathematics in teaching in the secondary school. The sophisticated vocabularies of secondary mathematics teachers is a somewhat unexplored territory and is therefore not widely reported in the literature. Chard's clear statement that "Vocabulary knowledge is as essential to learning mathematics as it is to learning how to read" (Chard n. d.) is indicative of the theme in the literature regarding the importance of words in the teaching and learning of mathematics. Vygotsky (1986: 107) states that "real concepts are impossible without words, and thinking in concepts does not exist beyond verbal thinking."

The idea that mathematics has its own specialised language has been explored in depth (Lakoff and Nuñez 2000; Devlin 2000). Tweed also succinctly points out that it is apparent from this literature that the idea of "*interweaving rich language and rich mathematics is not only a good idea, it is an essential ingredient in any successful mathematics learning. Use of language by students when doing mathematics will be enhanced when there are clear demands made on students to develop or create language that helps them solve problems and communicate the mathematics they learn and their solutions to problems.*" (Tweed n. d.).

Pimm (1991) regards 'mathematics as a language' but not a natural language like that of English. Mathematical language is defined as a 'mathematics register' which directly refers to the language used to express mathematical ideas and meanings. Pimm (1991) further emphasises that mental images in mathematics can be invented and controlled through the use of mathematical language. Kaput (1988) cited in Boulet (2007) also points out that mathematical language is not only a means of communication but also an instrument of thought which is the essence of mathematics.

The dated but seminal work of Pimm (1981: 139) states that "As teachers, our primary concern should be encouraging and improving the communication of mathematical meaning, both between teacher and pupil, but also among pupils themselves." This leads to the focus on the specific use of mathematical language by teachers in the process of teaching and learning of mathematics. Pimm (1981) mentions that the aim of the use of mathematical language is to help express, construct and communicate mathematical meanings. This inevitably becomes the responsibility of teachers who encourage their learners to become fluent in the oral and written language of mathematics. Boulet (2007) is of the opinion that in the field of mathematics education there is a clear awareness for the need to account for connections between language and mathematics. Boulet also states that in order for teachers to reveal the reasons behind mathematical procedures, it is imperative for them to make use of clear language.

Boulet's (2007) work encouraged teachers to prudently inspect the language that they make use of in the classroom. Boulet drew on the work of Raiker who claimed that problems in teaching and learning of mathematics can largely be attributed to spoken language. Raiker's (2002) work as cited in Boulet (2007: 10) confirmed that "teachers must be aware of the language they use when teaching mathematics and that the recommended vocabulary…should be used with caution." As a result Boulet (2007: 11) concluded that "it is not sufficient to tell teachers to be more sensitive to the language used in mathematical conversations", but that language frequently used in mathematical classrooms must be specifically addressed. A teacher's use of the correct mathematical language for understanding needs to be seriously considered since it is through words and language that we acquire concepts (Vygotsky 1986). Therefore, in contexts where the language of learning is not the mother tongue it makes sense that attention to detail of vocabulary may be necessary, and especially in senior grades where the language of mathematics is more sophisticated.

Two of Kilpatrick et al.'s (2001) strands of proficiency are also rooted in language. Conceptual understanding defined as *developing* mathematical ideas cohesively and efficiently, presupposes language with which to talk about mathematics. Adaptive reasoning, the promotion of logical thinking between concepts and situations, also implies being able to use language whether silent or spoken to justify procedures and to make mathematical choices.

Wanjiru and Miheso O-Connor (2015) used the Frayer Model integrated with technology to provide better opportunities for learners to understand their interaction with mathematics. They concluded using statistical measures that mathematics vocabulary instruction improved students' achievement in mathematics in Murang'a County, Kenya.

Much work has been done on language where a variety of languages in the classroom are resources that facilitate communication (Adler 2001; Setati 2008; Setati et al. 2009; Moschkovich 2009) This paper initiates a fine grained analysis of the language that teachers use in classrooms for communicating sophisticated concepts. Effective teaching is dependent on a bi-directional understanding between teachers and learners and authentic vocabulary is an essential prerequisite for these meaningful communications. This is confirmed by Riccomini et al. (2015: 235) who recognize that "Students' mathematical vocabulary learning is a very important part of their language development and ultimately mathematical proficiency." van Jaarsveld (2015, 2016, 2017) explains that 'exact' mathematical language uses correct vocabulary to describe mathematical objects. An expression like $\frac{x^2-x}{x}$ is often referred to as an equation that requires solving when in fact it is an expression that requires simplifying.

The authors above consider language either as a component, or language as a requirement for the development of and performance in mathematics. It is important however for teachers to be cognisant of the mathematical language that they use in their teaching – correct mathematical language is key to learners being able to formulate correct concepts and to articulate mathematical content. Current studies that examine exact teacher vocabulary are few and often deal with school mathematics in the earlier grades. These refer to language generally as a resource and provide strategies for teaching mathematical vocabulary. Exact mathematical vocabularies that teachers have are generally not a research focus. In contrast this paper shows that mathematical language and its vocabulary is an important aspect of rigour in and of itself to be learned in initial teacher education programmes and for experienced teachers in the field (van Jaarsveld 2016). Based on the literature, and findings of this paper it would seem that the 'content and language integrated learning' (CLIL) model used globally, where learners are not taught in their mother tongue, is worth implementing in the South African context. The cognitive-constructivist model espouses the 'co-construction of knowledge, of both content and language' (Marsh and Martín 2013). Baetens Beardsmore (2008) reports on the educational benefits of teaching in this way. The demand for English as a medium of instruction in South Africa (De Klerk 2002; De Wet 2002; Kgosana 2006; Rademeyer 2006; Uys et al. 2007) and internationally (Graddol 2010) makes the model an attractive option for implementation particularly in multilingual mathematics classrooms. This paper, therefore, focuses on the exactness of the mathematical language to preserve and promote the real mathematical meaning of words that are the basic constructs of concepts.

Language and Pedagogy

In order for teachers to excel at what they do they need to have a very deep and comprehensive understanding of the content being taught. Ball et al. (2008: 389) emphasise Shulman's (1986) point that mere content knowledge lacks pedagogical knowledge. Thus teachers have to have a deep level of content knowledge so that they are able to deconstruct content that is accessible and understandable for learners. This content knowledge becomes manifest in the language of thinking and teaching.

Together with content knowledge teachers have to know and understand when and how much of the content must be taught. Ball et al. (2008) talked about this knowledge as being curriculum knowledge. Besides content knowledge and curriculum knowledge, pedagogical knowledge is an important consideration. Ball et al. (2008) mentioned that teacher knowledge comprises this special domain of pedagogical content knowledge. This is said to be content knowledge that is unique to the teaching profession and is explained by Shulman (1986) cited in Ball et al. (2008) as being "the most useful ways of representing and formulating the subject that makes it comprehensible to others." Thus pedagogical content knowledge takes content knowledge and links it to the learner. Teachers therefore require a special knowledge of how to teach in order to make these links. 'Exact mathematical language' (van Jaarsveld 2016) is used in this paper as an example of special knowledge that teachers need in order to communicate meaningfully.

When Ball et al. (2008) stated that teaching mathematics entails a combination of things, namely; knowledge of mathematical ideas, skills in mathematical reasoning and eloquence in terms and examples, they recognise the importance of the language that teachers use.

van Jaarsveld (2016) focused on the need for teachers to be taught and to master "an exact mathematical language." He developed repertoires of language which sought to provide a structural framework to assist teachers in their thinking and teaching about specific mathematical objects. van Jaarsveld (2016: 12) found that these language repertoires "provided a structured system for assisting teachers in lesson preparation, lesson delivery and the development of exact mathematical language" and he showed that "the repertoires have their defining vocabularies that facilitate exact language use for speaking about mathematical objects." Teachers can therefore flexibly move between them depending on the nature of the mathematical object being taught. The language repertoires therefore also provide opportunity for varying pedagogical approaches.

METHODOLOGY

This mixed methods case study evaluated the extent to which a small sample of in-service teachers use exact mathematical language during teaching mathematics in a South African classroom. Mathematical language, within English as the language of instruction, was investigated using the following research questions.

1. To what extent do three South African in-service teachers use exact mathematical language?
2. To what extent are participating in-service teachers able to distinguish between four different language repertoires?
3. Do participating teachers value the use of exact mathematical language?

The mathematical language that participating teachers used in their classrooms was evaluated using the mathematical language repertoires which helped identify vocabular richness in relation to the four language repertoires; literal, algebraic, graphical and procedural (van Jaarsveld 2015, 2016, 2017). The beliefs held by teachers regarding the correct use of exact mathematical language is also described.

The sample group for this research project consisted of one novice teacher (two years teaching experience – Teacher B), one moderately experienced teacher (over five years of teaching experience – Teacher C) and one very experienced teacher (over ten years of teaching experience – Teacher A) all of whom taught Grade 10 and/or 11 mathematics.

The design of the research and the data collection sources are shown below. Data was collected in the order shown in the first column of Table 1.

Table 1: Data sources shown by shaded cells

	Teacher A	*Teacher B*	*Teacher C*
Lesson Observation			
Questionnaire			
Interviews 1 and 2			

Source: Authors

The research was conducted in a government school situated west of Johannesburg. The school was chosen because it was in a convenient geographic area. The data sets for this case study were derived from a questionnaire and two interviews. The questionnaire contained one closed-ended question as well as one open-ended question. Two of the participating teachers gave consent to completing the questionnaire. In question one (closed-ended question) of the questionnaire, teachers were first provided with a worked example showing how each of the language repertoires could be applied to a given mathematical object. This was provided to give teachers an idea of what was expected so that they could fairly interpret the object in terms of the repertoires. This contributed to the reliability of the questionnaire as each teacher was provided with the same information irrespective of their personal knowledge of the different language repertoires. The teachers were then asked to elaborate on one mathematical object using each of the four language repertoires. In question two (open-ended question) teachers were asked if they thought the use of exact mathematical language is important when teaching. In addition to this teachers were required to elaborate on their responses in order to gain some insights into their beliefs about the use of exact mathematical language.

Question one was analysed quantitatively against a master vocabulary list through which teachers' responses were classified under the following coding system; c = Correct, pc = Partially Correct, or i = Incorrect. Question one was also classified according to the number of correct and incorrect terms used in each repertoire by using a master memorandum. A comprehensive list of words/phrases pertaining to each repertoire to explain the mathematical object, $\sqrt{x} = \sqrt{2} - x$ was compiled. This was used to assess the density levels of the teachers' language used in answering question one of the questionnaire. Teacher responses to question two were open-ended and subjective.

The third data set comprised the transcriptions of two audiotaped interviews of up to 10 minutes. The purpose of the interviews was to gain clarity on participants' responses provided in the questionnaire. Interview questions were unstructured (McMillan and Shumacher 2010). Participants were asked to read out their responses on the questionnaire and were then invited to contribute further to what they had written. Because the transcriptions of the observed lesson were inadequate for analysis, teachers were requested to participate in a second interview on their use of exact mathematical language. The second interview comprised one structured and one semi-structured question (McMillan and Shumacher 2010). Participants were asked if they thought that one these topics (functions, algebra, Euclidean geometry, trigonometry, analytical geometry) was richer in language (more language dependent) than another. After they selected a topic each teacher was asked to brainstorm the vocabulary pertinent to it. Richness was defined as the vocabulary that the teachers used as a percentage of a comprehensive list (CL) of vocabulary associated with the mathematical object.

As the sample group used for this case study was small the findings cannot be generalised even though data were triangulated by the use of questionnaires and interviews.

Conceptual Framework

van Jaarsveld (2016) looked at the mathematical language of initial teacher education students. He defined a set of language repertoires that he suggested can be used by teachers in their lesson preparations, lesson delivery and the development of exact mathematical language. These mathematical language repertoires focus on speaking or thinking about mathematical objects in four different ways, namely: literally, algebraically, graphically and procedurally. Teachers can flexibly move between the different language repertoires based on the nature of the mathematical object.

Literal: Reading the object (symbolic notation) using the *correct* language and *in the correct sequence* where sequence matters.

Algebraic: Emphasises the *operations* that constitute the object.

Graphical or Cartesian: Focuses on interpreting the object as it is depicted in the *Cartesian plane.*

Table 2: Operationalising van Jaarsveld's (2016) four language repertoires for the mathematical object $x+y<6,\ x,\ y \in N_0$

Repertoires	*Formal definitions after van Jaarsveld (2016)*	*Word/terms that are appropriate for identifying each repertoire*
Literal	Reading mathematical objects or symbols in the right order	- *x* plus *y* - Less than 6 - *x* and *y* are whole numbers
Algebraic	Operations focused analysis	- Sum - Two whole numbers - Less than 6
Procedural	Algorithmic descriptions of how to arrive at a solution	- Standard form - Sketch, calculate coordinate pairs by substituting two different whole numbers *x* and *y*, reject intercepts because $x, y \in N_0$
Graphical	Cartesian plane as context of reference	Discrete relationship between variables *x* and *y* - Points, coordinate pairs that add to less than 6, - First quadrant - Beneath linear function

Source: Authors

Procedural or Algorithmic: The language associated with *reasoned explanations of how to 'do'* the mathematics (van Jaarsveld 2016).

Table 2 shows how the definitions above are operationalised when applied to the specific mathematical object $x + y < 6, \quad x, y \in N_0$ that was provided to participants as an example before they described the equation . Table 2 shows how each of the repertoires have their specific vocabularies.

RESULTS

All but one of the participating teachers did not differentiate between the language repertoires. This was revealed in their arbitrary use of mathematical terms to describe $\sqrt{x} = \sqrt{2} - x$. Although there was a presence of mathematical vocabulary it was inappropriately used. An 'equation' and an 'expression' was inferred as one and the same thing. These inaccuracies may be a function of a teacher's ontological path that emphasised a procedural view of mathematics rather than a conceptual one that would have required explanations of methods and justifications of solutions.

No teachers provided a vocabulary associated with the algebraic repertoire which describes the operations associated with the mathematical object. This may have arisen because mathematical objects are not contemplated at an algebraic level most likely because of its abstractness in comparison with manipulations that constitute the procedure of doing the mathematics. This repertoire entails the correct use of mathematical language in analysing the operations that make up a mathematical object which is an important aspect that learners need to be taught in order to develop deeper understandings of mathematical objects (van Jaarsveld 2016).

Although references were made to the Cartesian plane in the graphical descriptions of the object it was clear from the language used that teachers did not have accurate visualisations of the object. There was thus evidence of a graphical vocabulary but it was used incorrectly. Although teachers believed that mathematical language is important and that it had value for teaching and learning, their vocabulary lacked authenticity.

The participants generally had a mathematical vocabulary store with which to discuss mathematical objects but the vocabulary was often incorrectly used for its context.

These findings are discussed at length below.

DISCUSSION

In the first question of the questionnaire, teachers were asked to describe the object using the four language repertoires. A comprehensive list of words and phrases relevant to the equation appears in Table 3.

From Table 4 it is seen that only one teacher was able to describe $\sqrt{x} = \sqrt{2} - x$ correctly in terms of the four language repertoires which may have been a result of the teachers' unfamiliarity with the language task.

Literal Repertoire

Teacher A gave a correct response while Teacher C gave a partially correct response for the literal description of the mathematical equation. Teacher A used the word, 'minus' and Teacher C used 'subtract'. Typically 'minus' indicates the sign of a term, where 'subtract' would lay emphasis on the operation between terms. 'Minus', although commonly used, is according to $\sqrt{x} = \sqrt{2} - x$ van Jaarsveld's (2016) algebraic language repertoire, technically incorrect since according to his definition the equation was not interpreted in terms of the operations on the variable x .

Algebraic Repertoire

Both teachers made use of the word *'difference'* in their algebraic repertoire. However, the operations associated with the variable were not conveyed. Instead Teacher A reverted to what may be a more elaborate literal explanation in saying *'the expressions on the right hand side (RHS)'* needed to be *'equal to the root of x'*. Teacher C made reference to *'the square root of x'* and the *'square root of two and x'*. Considering these responses in relation to $\sqrt{x} = \sqrt{2} - x$, right hand side (RHS) contains either a single expression, $\sqrt{2} - x$, or two terms, $\sqrt{2}$ and $-x$. There is therefore a loose use of language in referring to *'the expressions RHS'* since there is essentially only one. Teacher C's reference to the RHS being the *'square root of two and x'* implies the sum of the terms rather than their difference. Because of the mathematically incorrect use of 'and' it is evident that the algebraic and literal language repertoires are conflated.

Table 3: Comprehensive List (CL) of terms per repertoire that could have been used to describe the mathematical object $\sqrt{x}=\sqrt{2}-x$ used in the questionnaire

Literal	*Algebraic*	*Graphical*	*Procedural*
- Square root of x, - Equal - Square root of 2, - Minus/subtract x	- Equation in x - Solve - Additive inverse - A real number x - Increased by $\sqrt{2}$ - Equal - Square root of some real number	- x value(s)/domain - Number line - Positive branch - Inverse parabola $y=\sqrt{x}$ - Cuts/intersects - Decreasing at a constant rate - Linear - Function - $y=\sqrt{2}-x$ - Solution - Equation - Point of intersection - Two functions - x value on the number line, vertically below it.	- Solving for x - Squaring both sides - Equation - Reduce - Surd - Extraneous roots - Added to both sides - Quadratic - Squared $\left(x^{\frac{1}{2}}\right)^2$ - Divide - Denominator - Fractional - Exponent - Numerator - Squaring a Binomial - Trinomial - $2-\sqrt{2}x+x^2$ - Standard form - Factorised - Formula for the roots of a quadratic equation - $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ - Derived - Completing the Square - General form of a quadratic equation $ax^2+bx+c=0$ - $a=1$, $b=\sqrt{2}$ and $c=2$ - Pattern of equivalence - Visualisation - Cartesian plane - One solution - Positive - Calculated - Rounded off - Number of decimal places - Approximation - Root - Exact value of x
Total terms: 4	Total terms: 7	Total terms: 16	Total terms: 37

Source: Authors

Procedural Repertoire

Both teachers' responses were partially correct compared with the CL. Teacher A said that the *'equation'* had to be *'rewritten'* first in order to *'identify the subject'* followed by wanting to *'simplify the equation'*. The steps to this *simplification* were not given. Teacher A proceeded to say that simplification would give rise to *'a parabolic graph'* with *'possibly two solutions'* together with mentioning a *'validation test'* that must be carried out. Conventionally

Table 4: Summary of teachers' responses to question one of the questionnaire using the coding system referred to in Table 3 (c = correct, pc = partially correct, i = incorrect)

	Teacher A	*Teacher C*
Literal	c	pc
Algebraic	i	i
Graphical	pc	pc
Procedural	pc	pc

Source: Authors

equations are solved and not simplified and the lack of regard for the distinctly different instructions is an indicator of the absence of an exact mathematical language. Besides the language considerations the initial instruction to rewrite the equation to identify its subject is problematic in that the subject is subsumed in $\sqrt{x}$ and *(-x)*. Should the equation have reduced to a perfect square equal to zero the instruction would have made sense.

Teacher C initially gave an explanation of the relationships between the variables and then made mention that the *'object'* has to be *'written as* $x=\left(\sqrt{2}-x\right)^2$ in order to make it *'easier to solve for x'*. The teacher then only addressed the expression on the *'right hand side'* by explaining that it *'can be expanded to form a quadratic equation'*. Teacher C also indicated that *'there will be two solutions'* and also referred to testing these solutions by *'substituting both into the original equation to find which root satisfies the equation'*. Here again if the RHS is expanded it results in a quadratic expression, not an equation. This account indicates that there is some knowledge of what is required for a procedural explanation albeit a skeletal account of the procedures, without reasons and the incorrect reference to equation.

Graphical Repertoire

Teachers' responses to the graphical explanation of the equation were also identified as being partially correct on the basis that each teacher responded in a very general way instead of being specific regarding the mathematical object $\sqrt{x}=\sqrt{2}-x$. Teacher A said that the *'equation'* would *'result with co-ordinates on a quadratic graph'*, together with mentioning that *'two valid points could be the root of the equation'*. This is a further instance of the conflation of two repertoires, in this case the procedural and graphical. The *'could be'* suggests uncertainty and an inability to visualise that the positive branch of the inverse parabola intersects only once with the decreasing linear function for the equation to result in a single valid solution. The reference to the 'coordinates on a quadratic graph' shows the presence of vocabulary but also shows the arbitrary usage of words associated with the Cartesian plane. Although the vocabulary appropriately relates to the graphical repertoire it does not convey the true graphical meaning of the equation. The solution is actually the value of the coordinate of the point of intersection of the two graphs, not its coordinate pair. The reference to 'two valid points' as solutions to the equations is clearly incorrect.

Teacher C said that *'this equation should form a parabola'* and that its *'shape'* would be *'dependent on whether the coefficient of its leading term is positive or negative'*. The inference that the equation should form a parabola is incorrect since the equation illustrated in the Cartesian plane is the positive branch of the inverse parabola intersecting the linear function. Neither teacher attempted to provide a sketch of the equation in the Cartesian plane. It was however not requested of them to do so. This is another case of the conflation of repertoires, in this case procedural/algebraic and graphical through the use of 'equation'/'coefficient' and 'parabola'.

The density of the vocabulary associated with $\sqrt{x}=\sqrt{2}-x$ is found in Table 5.

In the last column of Table 5 'density of teacher language' is defined as the percentage of the comprehensive list of words/phrases that teachers used. For the literal repertoire, teachers made use of all the terms in the comprehensive list. The teachers' language density for this repertoire was therefore one hundred percent. Even though Teachers A and B obtained seventy-one percent and fifty-seven percent respectively for the presence of an algebraic repertoire it occurs along with incorrect descriptions. Teachers were therefore not able to apply their vocabulary accurately to produce an adequate algebraic explanation of the mathematical object.

The lower than expected density in the procedural repertoire, which is generally strongest, may be related to the complex variety of strategies involved in solving the equation, $\sqrt{x}=\sqrt{2}-x$, as detailed in Table 3.

The graphical repertoires of the teachers are similarly less dense alongside partially correct vocabularies. It indicates that these teachers are

Table 5: Vocabulary density levels for the equation $\sqrt{x} = \sqrt{2} - x$ (See Table 3 for associated vocabulary)

Teacher and codes as in Table 3. (c = correct, pc = partially correct, i = incorrect).	*Teaching experience*	*Number of terms used per language repertoire by teachers A and C*	*Number of possible associated terms (CL)*	*Density of teacher vocabulary defined as the vocabulary used as a % of a comprehensive list of an associated vocabulary*
Literal				
Teacher A (c)	Highly experienced	4	4	100
Teacher C (pc)	Moderately experienced	4	4	100
Algebraic				
Teacher A (i)		5	7	71
Teacher C (i)		4	7	57
Procedural				
Teacher A (pc)		8	37	22
Teacher C (pc)		9	37	24
Graphical				
Teacher A (pc)		6	16	38
Teacher C (pc)		6	16	38

Source: Authors

unfamiliar with a graphical repertoire. The partially correct code alongside this is indicative of teachers switching between repertoires.

The analysis above shows that the teachers have a mathematical vocabulary to draw from in order to assist them in talking about mathematical objects from the literal and algebraic repertoires. They however lack density in their mathematical vocabulary for use under the procedural and graphical repertoires. It may be appropriate to suggest that working with a mathematical object from the perspectives of the four language repertoires, teachers could gain deeper insights about the object being worked with and teach comprehensively. As such this would have positive implications for their lesson preparations, lesson delivery, and development of exact mathematical language for teaching and learning as suggested by van Jaarsveld (2016).

In the second question of the questionnaire, teachers were asked if they thought the use of exact mathematical language is important when teaching (See Table 6).

These responses by Teacher A and C indicate that they do value the use of exact mathematical language when teaching. Teacher A responded by stating *'definitely'* while Teacher C says that it *'is important'*. Teacher A responded by stating that *'math is a language... free from the rules and regulations that govern our normal language'* while Teacher C said that *'each subject… should have its own language of instruction'* which concurs with Pimm (1991).

Teacher A indicated a connection between the use of correct mathematical language and concepts by stating that its use *'would aid in the understanding of concepts and clearly define its perimeters'*. Likewise this theme of lan-

Table 6: Teachers' views on the importance of mathematical language for teaching

Teacher A	*Teacher C*
"Definitely. Math is a language on its own, free from the rules and regulations that govern our normal language. The use of correct mathematical language would aid in the understanding of concepts and clearly define its perimeters."	"The use of exact mathematical language when teaching is important for the following reasons: now, it relates to a particular concept or concepts and results in easy problem solving; each subject is unique and so should have its own language of instruction; each topic has a direction to be followed, and so language plays a vital role if comprehension is to be achieved."

Source: Authors

guage and concepts was identified in Teacher C's response that the correct use of mathematical language would *'relate to a particular concept or concepts… resulting in easy problem solving'*. Furthermore Teacher C stated that *'language plays a vital role if comprehension is to be achieved'* and that *'appropriate language leads to better understanding of a concept'* as stated by Pimm (1981) that the aim of the use of mathematical language is to help express, construct and communicate mathematical meanings.

During the second interview participants were first asked if they thought that one particular topic (functions, algebra, Euclidean geometry, trigonometry, and analytical geometry) was richer in language (more language dependent) than another. Teacher A chose Functions while Teachers B and C chose Euclidean geometry.

The second question asked the teachers to brainstorm vocabulary they would use to teach one lesson from their chosen topic that had particular relevance for the learning of the topic. For example Teacher A recalled *'linear functions'* and *'quadratic functions'*, while Teacher B thought about *'congruency'* and *'theorems'* both inter alia words. The graphical repertoire for Teachers B and C was not applicable here because they chose Euclidean Geometry. In total Teacher A recalled thirty-five mathematical words/phrases, Teacher B recalled thirty-four mathematical words/phrases and Teacher C one word/phrase. The apparent ability of the teachers to brainstorm vocabulary indicates that they have a vocabulary store to draw on.

Figure 1 shows that no recalled words/phrases used by any of the teachers could be classified under the algebraic repertoire. In contrast to this, all three teachers provided words/phrases that could be classified for use under the graphical repertoire albeit incorrectly used in their descriptions.

CONCLUSION

To answer the research questions, the participating teachers demonstrated that they have

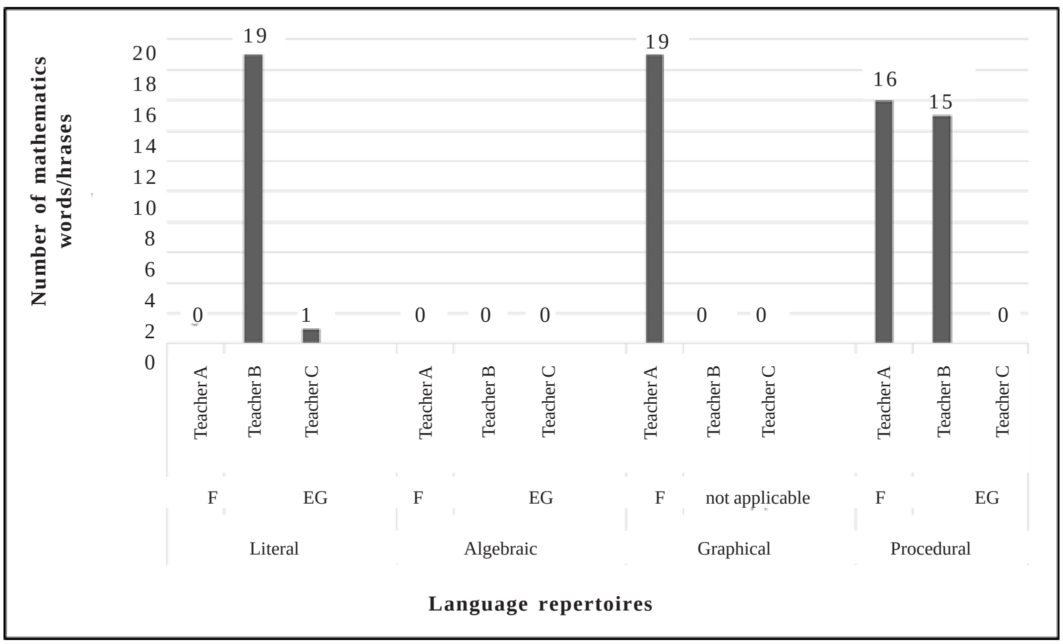

Fig. 1. Bar graph showing the classification and number of words recalled by each teacher per repertoire per chosen mathematics topic during a second interview. (F = Functions, EG = Euclidean Geometry). Note that a graphical repertoire would not be applicable in Euclidean Geometry unless the content was interpreted from the perspective of analytical geometry
Source: Authors

a mathematical vocabulary but that despite their belief in the importance of a mathematical language for teaching, their vocabularies let them down in describing $\sqrt{x}=\sqrt{2}-x$ using the language repertoires.

The presence of a mathematical vocabulary does not presuppose that it will be authentically applied in teaching. The mathematical language repertoires provide a conceptual tool for both teachers and researchers for self-examining their mathematical fluency and for assessing the authenticity of mathematical language used in teaching respectively. An authentic vocabulary also has the benefit of developing a conceptual understanding demonstrated in being able to fully describe, discuss and reason about mathematical objects. The benefits accrue to teachers and learners alike.

RECOMMENDATIONS

In order to help teachers use an exact mathematical language for the betterment of teaching and learning, workshops for in-service teachers that focus on developing teachers' fluency in and command of the four language repertoires may be beneficial. These workshops should aim to help teachers talk, think and work with mathematical objects for authentic classroom communication. This may also provide for more meaningful teaching that may help conceptual understanding for teachers and learners. A vocabular approach to classroom mathematics as confirmed by the participants in this study has value in that it has potential for promoting both mathematical content and pedagogical knowledge.

In a multilingual South Africa where English is the primary language of instruction the value of implementing a content and language integrated learning approach which espouses learning a second language through addressing the vocabulary of the subject content, mathematics in this case, has manifold benefits.

Research on the impact of exact mathematical language on teaching and learning is relatively unexplored and therefore warrants and invites investigation. This is particularly so in the secondary grades where mathematical concepts are tied up in sophisticated vocabularies.

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